



**Theory, Dynamics and Applications of**

# **Magnetic Resonance Imaging- I**

Edited by Omotayo Bamidele Awojoyogbe



# Theory, Dynamics and Applications of Magnetic Resonance Imaging-I

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## **Editor's Introduction**

There can be few better examples of the complex and unanticipated interactions of basic research and technological innovation than the development of magnetic resonance imaging (NMR/MRI) techniques for multidisciplinary research. The method has the rather unusual and attractive features that it is totally non-destructive and non-invasive and for these reasons it has interesting applications in almost all fields of research.

Nuclear magnetic resonance imaging (NMRI) is an important modality of medical imaging. NMRI uses magnetic fields to manipulate magnetization in a way that makes it a conveniently measurable signal which encodes spatial location and density information. Physically, this is a complicated process which relies on effects at both the quantum and macroscopic levels. Mathematically, with correctly designed sequence of magnetic field applications, the recorded signal can be extensively explored for maximum applications.

The image is essentially the distribution of water molecules in a tissue slice of the patient. Since tissue densities have varying water contents and NMR has very high contrast, it is possible to obtain detailed images of internal soft tissue. The advantage of this method is that it is very good at contrasting soft tissue since it primarily measures water content of tissues with no ionizing radiation. The disadvantages are that it is expensive, it has long scan times and poor imaging of bone tissues, and it is too slow to image dynamic processes at high resolution.

Nuclear magnetic resonance is a physical phenomenon which is based on the magnetic property of an atom's nucleus. All nuclei that contain odd numbers of nucleons and some that contain even numbers of nucleons have an intrinsic magnetic moment. Hydrogen nuclei, fluorine, carbon-13 and oxygen-17 all have distinctive magnetic properties that make them suitable for NMR studies.

First introduced in the 1940's by Felix Bloch and Edward Purcell to measure the magnetic moment of nuclei in liquids and solids, NMR is based on protons in a nucleus having an intrinsic spin angular momentum, thus a magnetic moment. When a constant magnetic field is applied, a nucleus will resonate like a mechanical oscillator when driven in to an excited energy state by an electromagnetic (EM) wave (in the radio frequency range for NMR) at the correct frequency, which is determined by the strength of the magnetic field and the magnetic moment of the nucleus. Traditionally, NMR was done by sweeping the magnetic field strength while applying a continuous EM wave and measuring where the signal from the nuclei emitted peaked (continuous wave NMR, or CW NMR). However, modern NMR is usually done by applying a pulse of rf wave (pulsed NMR), which contains a broad spectrum of frequency components, and measuring the radio signal, termed the free induction decay (FID) as the signal decays like a damped oscillator, emitted from the nuclei after the pulse. Using a Fourier transform of the FID signal, the resonance frequency of the nuclei can be determined. Moreover, pulsed NMR can provide information concerning the physical properties of the nuclei measured. One of the very important properties of NMR/MRI is the relaxation times, or the time required for the nuclei in the sample to return to their ground state after being saturated by a strong EM pulse of the sample. These parameters

make MRI unique and vast with many applications in the fields of science, medicine, engineering and agriculture.

Nuclear magnetic resonance is inherently a three-dimensional phenomenon. The spatial resolution of a three-dimensional set of data is usually equal in all three directions. At the core of the NMR, a magnetic resonance imaging (MRI) makes use of the fields dependency on the precession frequency by superimposing a magnetic field gradient onto the static polarizing field  $H_0 = H_z$  to spatially encode information into the rf (radio frequency) signal. With this three-dimensional data in hand, surfaces can be detected mathematically. A computer translates these signals into highly detailed cross sectional images. The images are essentially maps of the locations of hydrogen in the body.

The sensitivity of nuclear magnetic resonance to molecular structure has made it a valuable research tool in organic chemistry, enabling chemists to determine hydrogen locations in crystals, something that cannot be done using X-ray diffraction. Nuclear magnetic resonance has also been used to study electron densities, chemical bonding, the compositions of mixtures, and to make purity determinations.

The basic requirements for NMR spectroscopy are that the magnetic field be homogenous over the volume of the sample, that there be a radio frequency field rotating in a plane perpendicular to the static field, and that there be a means of detecting the interaction of the frequency field with the sample.

Techniques that have been developed for the observation of NMR signals fall into two categories: pulsed and continuous wave. In the case of the pulsed methods, an applied rotating (or alternating) magnetic field

with a frequency at or near the Larmor frequency (that is, frequency of precession) of the nucleus to be studied is directed at right angle to the static field. If the rotating field is applied at exact resonance, the nuclei precess about that field as though there was no static field. Continuous wave methods are either high resolution or broad line. Broad line widths are produced by most oriented molecules exhibiting strong magnetic dipolar interactions, so broad line spectroscopy does not permit measurements of chemical shifts and spin-spin coupling. High resolution on the other hand, has been used to identify molecules, to measure the electronic effects, to determine structure, to study reaction intermediates, and to follow the motion of molecules or groups of atoms within molecules. For high resolution studies, the magnetic field must be uniform to 1 part in  $10^8$  for a 100MHz instrument, if a resolution of 7MHz is to be obtained. In the case of broad line studies, 5 parts in  $10^6$  may be adequate.

Nuclear magnetic resonance has been used to study the physics and chemistry of solids, including metals, semiconductors, magnetic solids and organic materials. Physical phenomena that can be studied by NMR include conduction-electron paramagnetism; spin waves and magnetic fluctuations in ordered magnetic materials; metal molecular transitions; charge density wave phenomena; spin-freezing in spin glasses and frequency shift and spin-lattice relaxation effects. At low temperatures, NMR has been used to make temperature measurements and to study the super fluid phases of  $^3\text{He}$ .

In the fields of organic chemistry and materials science, NMR has been used to study polymers, amorphous systems and complex molecular solids. In many of these systems, the NMR line widths of the

nuclei are dominated by dipolar fields arising from neighbouring magnetic moments. These systems exhibit complex NMR spectra due to shifts in nuclear magnetic resonance frequencies.

In the case of complex molecules in liquid environments, the molecules undergo a tumbling motion, producing very sharp NMR spectra. The technique used to study these systems is known as Fourier transforms NMR Spectroscopy.

Specifically, nuclear magnetic response is extremely useful for analyzing samples non-destructively. Radio waves and static magnetic fields easily penetrate many types of matter and anything that is not inherently ferromagnetic. For example, various expensive biological samples, such as nucleic acids, including RNA (Ribonucleic Acid) and DNA (Deoxyribonucleic Acid), or proteins, can be studied using nuclear magnetic resonance for weeks or months before using destructive biochemical experiments. This also makes nuclear magnetic resonance a good source for analyzing dangerous samples.

Nuclear magnetic resonance has been used in data acquisition in petroleum industry for petroleum and natural gas exploration and recovery. A borehole is drilled into rock and sedimentary strata into which nuclear magnetic resonance logging equipment is lowered. Nuclear magnetic resonance analysis of these boreholes is used to measure rock porosity, estimate permeability from pore size distribution and identify pore fluids (water, oil and gas).

NMR is also a tool used not only in well logging, but also in petrophysics. Additional applications of surface NMR (SNMR or magnetic resonance sounding - MRS) have shown the power of the

method for hydrogeophysical purposes. The power of the NMR method relies on the fact, that it is the only geophysical method which delivers direct information of the water content in samples or the subsurface. Moreover, it allows the determination of important structural parameters like porosity, surface/volume ratio or hydraulic conductivity.

Recently there is an increasing demand for reliable information of the water content and structural parameter like water retention capabilities of soils. For applications on the meter and decimeter scale – like soil physics, precision farming (agrogeophysics) or dam quality and stability investigations – NMR could therefore be a valuable instrument in the fields of agriculture and geophysics.

Nuclear magnetic resonance has also entered the arena of real-time process control and process optimization in oil refineries and petrochemical plants. Two different types of NMR analysis are utilized to provide real time analysis of feeds and products in order to control and optimize unit operations. Time-domain NMR (TD-NMR) spectrometers operating at low field (2-20MHz for  $^1\text{H}$ ) yield free induction decay (FID) data that can be used to determine absolute hydrogen content values, rheological information, and component composition. These spectrometers are used in mining, polymer production, cosmetics and food manufacturing as well as coal analysis.

Volume I, of this book series presents the *Bloch phenomenological equation*, which provides a model for the interactions between applied magnetic fields and the nuclear spins in the objects under consideration. It analyzes macroscopic averaged models that describe the interaction of aggregates of spins, with applied magnetic fields. This is an integrated science text in the direction of promoting a long-term future filled with

important developments in quantitative, theory-based biology towards addressing current disadvantages of MRI. The goal and intention is to see exactly diseased conditions at quantum (molecular) level, in order to have thorough understanding of their specific causes (or how they are caused), trace and monitor their progression and get the best treatment for them. This book is another effort to break down the traditional boundaries between mathematics, physics, chemistry, and biology, and does so in a compelling fashion.

The audience for whom the book is intended is mathematical scientists, biomedical physicists, biomedical engineers, geophysicists and computer scientists. The book can be useful to people that do research in the field of NMR/MRI. It will be more useful to the beginners or scientists that want to see the connection between medical biophysics and mathematics. The book is relevant to current research trends and may advance a beginner's ability to do research in medical MRI physics. The book is of great usefulness for all those persons who want to develop a carrier in MRI.

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By  
The Editor  
*Omotayo Bamidele Awojoyogbe*

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